Subatomic Physics
2022-2023

Exam

Monday 30 January 2022 15:00-17:00 CET

Remarks and instructions

- Please write your name and your student number on every sheet
- Please put consecutive page numbers on your pages
- Provide your answers with clear context and explanations
- There are 13 points and 1 bonus point(s). The grade is 1+9*min(number of points,total)/total
- The amount of points is listed in front of each sub-question.

1 General concepts (3.5 points)

Please keep your answers to a maximum of three sentences.

a) (1.0) Discuss briefly what "size" means for fundamental particles and how this differs from the size of composite particles.

b) (1.0) The electromagnetic interaction is considered a long range interaction. On the other hand, the weak and the strong force are both short range, although they are so for markedly different reasons. Describe how this comes about for both cases.

c) (0.5) How come we have never observed quarks directly? Clarify which quantum number causes this in combination with the properties of the strong force and what it means for the propagation of particles interacting through the strong interaction.

d) (1.0) To detect subatomic particles, it is needed to transform a microscopic interaction to a macroscopic effect. Describe one such process, going from the type of material interaction to detection.

2 Nuclear decay (2.5 points)

The Nitrogen isotope ${}^{16}_{7}\mathrm{N}$ has a half-life of about 7.1 s and decays to ${}^{16}_{8}\mathrm{O}$ which has a mass of about 15.9949 u.

a) (1.0) What type of decay is this and which interaction(s) is/are responsible? Write down the reaction mechanism of the aforementioned decay.

b) (0.5) Name a model to estimate the mass of ${}^{16}_{7}\mathrm{N}$ and other nuclei.

c) (1.0) What is the maximum energy of the electron released by this decay, given that the mass of ${}^{16}_{7}\mathrm{N}$ is about 16.0061 u?

3 Mesons (4.0 points)

One type of hadronic matter comes in the form of $q\bar{q}$ pairs, called mesons.

a) (1.0) The π^0, the lightest meson, is composed of a linear combination of the two lightest $q\bar{q}$ pairs, $u\bar{u}$ and $d\bar{d}$, and decays predominantly to $\gamma\gamma$. Which interaction is responsible and why can the π^0 not decay to two gluons?

b) (1.0) Why is the lifetime of the π^+, which consists of $u\bar{d}$ and decays predominantly through $\mu^+\nu_\mu$, nine orders of magnitude higher than π^0? Would this ratio of lifetimes be different if the masses of the pion states were much higher?

c) (1.0) The η meson has spin-0 and is also a $q\bar{q}$ state, like the pion, but is a linear combination of three components (including also the $s\bar{s}$ state). It decays to $\gamma\gamma$ as well as to $\pi^0\pi^0\pi^0$. What does that imply for its parity and C-parity?

d) (1.0) Can the η meson decay to $\pi^+\pi^-$ as well (assuming it is a strong decay)? Motivate your answer by calculating the parity and C-parity of this final state and indicate how they determine the possibility of this decay.

4 NA62 experiment (3.0 + 1.0* points)

At CERN, in the North Area (NA), the NA62 experiment is searching for the decay $K^+ \to \pi^+\nu^+\nu^-$. This decay proceeds through loop diagrams involving the weak decay, such as the one shown in Figure 1. Two years ago, NA62 observed 20 events in a sample where they expected 7 background events and reported a measurement of the signal branching fraction of $\mathcal{B}(K^+ \to \pi^+\nu^+\nu^-) = (10.6^{+4.0}_{-3.4}(\text{stat.}) \pm 0.9(\text{syst.})) \times 10^{-11}$, where the first uncertainty is statistical and the second is systematic.

The charged kaon, K^+, is a particle composed of an up-quark and an anti-strange quark, which decays with a lifetime of $(1.23 \pm 0.08) \times 10^{-8}$ s, and is produced for the NA62 collaboration by colliding 400 GeV/c protons on a beryllium target. In addition to the signal decay $K^+ \to \pi^+\nu^+\nu^-$, the K^+ also decays through the process $K^+ \to \pi^+\pi^0$, which has a measured branching fraction of $(20.67 \pm 0.08)\%$.

a) (1.5) Draw the interaction diagram of the $K^+ \to \pi^+\pi^0$ decay. Which CKM elements are involved in both decays? Note that there are three contributions participating in the $K^+ \to \pi^+\nu\nu$ decay, one for each up-type quark. Based on the CKM elements involved, what size would you expect the branching fraction of $K^+ \to \pi^+\pi^0$ to be relative to the $K^+ \to \pi^+\nu\nu$ decay? If you find a difference compared to the expected value, explain where this could come from.

b) (0.5) As the goal is to detect the process $K^+ \to \pi^+\nu^+\nu^-$, give a detection principle that could be used to separate K^+ particles that did not decay in the detector from those that decayed following $K^+ \to \pi^+\nu^+\nu^-$. Note that the K^+ particles entering the NA62 detector have a well-defined momentum.

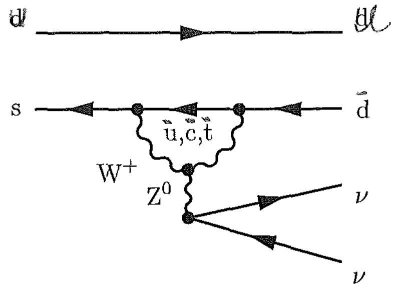

Figure 1: One of the interaction diagrams for the $K^+ \to \pi^+\nu^+\nu^-$ decay

c) (1.0) The more common mode $K^+ \to \pi^+\pi^0$ can also act as a background and has to be rejected. The π^0 commonly decays to a pair of photons. Indicate a detector principle that can be used to identify this background.

d) To measure the small branching fractions associated with the $K^+ \to \pi^+\nu^+\nu^-$ decay, an enormous number of charged kaons needs to be produced. Assuming that the beryllium target has a density of 1.85 g/cm^3, a thickness of 0.4m, an atomic mass of 9.012 u, and that the cross section for 400 GeV/c protons on this target is 1 mb, calculate the required number of protons hitting the target per second to achieve a production of 45 million charged kaons per second.

e) (1.0*) One possibility to modify the branching fraction of the $K^+ \to \pi^+\nu^+\nu^-$ decay is through the exchange of a particle directly coupling neutrinos to quarks, with a so-called leptoquark. Draw the first order Feynman diagram for the decays in question via a leptoquark.

Formulas and constants

Constants

Speed of light	c	$3.0 \cdot 10^8$	m/s
Planck constant	h	$4.1 \cdot 10^{-24}$	GeV s
Electron mass	m_e	0.51	MeV/c^2
Proton mass	m_p	938.27	MeV/c^2
Neutron mass	m_n	939.57	MeV/c^2
Pion mass	m_π	139.57	MeV/c^2
Muon mass	m_μ	105.66	MeV/c^2
Kaon mass	m_{K+}	493.68	MeV/c^2
Atomic mass unit	u	931.49	MeV/c^2
Avogadro's constant	N_A	$6 \cdot 10^{23}$	mol^{-1}
Barn	b	$1 \cdot 10^{-28}$	m^2

Relations and models

De Broglie wavelength

$$\lambda = \frac{h}{p} ,$$

where p is momentum and h is Planck's constant.

Relativistic mechanics

$$E^2 = p^2c^2 + m^2c^4$$

$$E = \gamma mc^2; \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Decay

$$N(t) = e^{-t/\tau} ,$$

where t is proper-time and τ is the lifetime of the particle.

Cherenkov radiation

The angle θ of the Cherenkov light cone is given by

$$\cos(\theta) = \frac{1}{n\beta} ,$$

where n is the refraction index and β is the speed of the charged particle relative to the speed of light in vacuum

Charge parity

$$\hat{C}\left|f\bar{f}, J, L, S\right\rangle = (-1)^{L+S}\left|f\bar{f}, J, L, S\right\rangle ,$$

where f are fermions, L and S are orbital and spin angular momentum

CKM matrix and Wolfenstein parametrization

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} ,$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) ,$$

where A, λ, ρ and η are dimensionless parameters and terms are expanded up to order $\mathcal{O}(\lambda^3)$ Their values are roughly 0.8, 0.23, 0.14 and 0.35, respectively